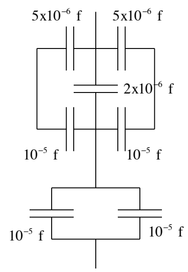
Problem 16.42

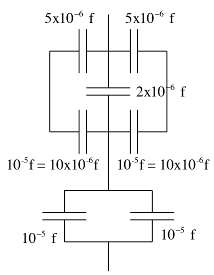
a.) Find the equivalent capacitance for the circuit to the right:



b.) What charge is stored on either of the 5 microfarad cap if the voltage across the entire system is 60 volts?

(This is a hard problem. Don't take a lot of time on it. It is primarily designed to make you think about the relationships between capacitance, charge, and voltage as they are related to series and parallel combinations of capacitors.)

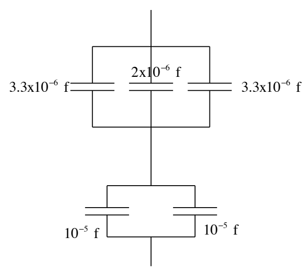
a.) Find the equivalent capacitance for the circuit to the right:

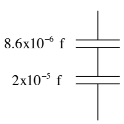


(note that the two caps in each upper side-shoot are in series)

$\Rightarrow$

(note that the two caps in the lower section are in parallel)





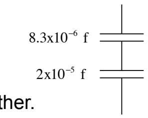
$\Rightarrow$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{8.6\text{x}10^{-6}\text{ f}} + \frac{1}{2\text{x}10^{-5}\text{ f}}$$

$$\Rightarrow \quad C_{eq} = 6.01\text{x}10^{-6}\text{ C}$$

b.) What charge is stored on either of the 5 microfarad caps if the voltage across the whole combination is 60 volts?



We have two groups of capacitors with each group being in series with the other. Taking the equivalent cap for each group, that combinations can be represented as shown to the right. We know that the 60 volts will be distributed between these two equivalent caps, and that because they are in series, the charge on each will be the same. With that, we can determine that charge on each as follows:

$$V_o = V_1 + V_2$$

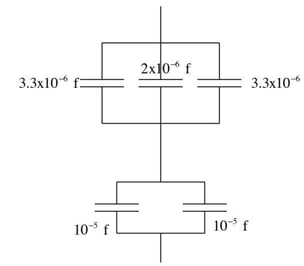
$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$60\text{ volts} = \frac{Q}{8.6\text{x}10^{-6}\text{ f}} + \frac{Q}{2\text{x}10^{-5}\text{ f}}$$

$$\Rightarrow \quad Q = 3.6\text{x}10^{-4}\text{ C}$$

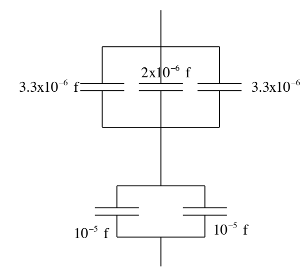
This is the charge on each of the two equivalent capacitances in the sketch above (they are in series, so there is the same charge associated with each). If we can determine how that charge is distributed between the actual capacitors making it up the combination, we are done.

To that end, I've redrawn the circuit replacing each upper-side branch with its equivalent capacitance.



Why have I done this? Because if we can determine how much charge is associated with the $3.3\ \mu f$ equivalent capacitance, we will know the charge on each of the actual caps making up that section including that of $5\ \mu f$ cap .

From our previous work, we know the net charge distributed across the three branches making up the upper section is $3.6\text{x}10^{-4}\text{ C}$.



Looking at the upper section, the voltage V across each section is the same (they are in parallel). We can see that the bigger the equivalent capacitance, the more the charge will be associated with that branch (q and C are proportional). So if we take the total capacitance, determine what fraction of capacitance is associated with the $3.3\ \mu f$ branch, then multiply that fraction by the total charge to be shared between the three branches. Doing this, we get:

$$Q_{3.3} = \frac{C_3}{C_{total}} Q_{total}$$

$$= \frac{3.3\text{x}10^{-6}\text{ f}}{8.6\text{x}10^{-6}\text{ f}}\left(3.36\text{x}10^{-4}\right)$$

$$= 1.29\text{x}10^{-4}\text{ C.}$$